Solution – III

Question 1

a) $\mu = \sum xf(x) = 0(0.08) + 1(0.20) + 2(0.19) + 3(0.24) + 4(0.14) + 5(0.13) + 6(0.02) = \mathbf{2.63}$

b) $\sigma^2 = \sum x^2 f(x) - \mu^2 = \begin{bmatrix} 0^2(0.08) + 1^2(0.20) + 2^2(0.19) + 3^2(0.24) \\ +4^2(0.14) + 5^2(0.13) + 6^2(0.02) \end{bmatrix} - (2.63)^2 = 2.4131$

$$\boldsymbol{\sigma} = \sqrt{2.4131} = \mathbf{1.553}$$

c) $\mu - \sigma = 2.63 - 1.553 = 1.077$

$\mu + \sigma = 2.63 + 1.553 = 4.183$

The interval $[1.077, 4.183]$ **includes** the *x*-values 2, 3, and 4.

$P[1.077 \leq X \leq 4.183] = f(2) + f(3) + f(4) = \mathbf{0.57}$

d) $2\sigma = 3.106$

$\mu \pm 2\sigma = 2.63 \pm 3.106 \; or \; [-0.476, 5.736]$

The interval $[-0.476, 5.736]$ includes all the *x* values except 6, so

$P[-0.476 \leq X \leq 5.736] = \mathbf{0.98}$

Question 2

a) The probability that the student will get either of the two winning tickets is

$$\frac{2}{1000} = \mathbf{0.002}$$

b) Consider X = dollar amount of the student's winnings. The random variable X can have the values 0 or 200 with probabilities 0.998 and 0.002, respectively.

x	$f(x)$	$xf(x)$	
0	0.998	0	
200	0.002	0.4	
		0.4	$= E(X)$

Considering now the purchase price of \$1, the student's

expected gain = \$0.40 − \$1 = $-\$\mathbf{0.60}$, that i s, expected loss = $\$\mathbf{0.60}$

Question 3

a) $P[Z > 0.62] = 1 - P[Z < 0.62] = 1 - 0.7324 = \mathbf{0.2676}$

b) $P[-1.40 < Z < 1.40] = P[Z < 1.40] - P[Z < -1.40] = 0.9192 - 0.0808 = \mathbf{0.8384}$

c) $P[|Z| > 3] = P[> 3] + P[< 3]$

$= 2P[Z < -3]$ (by symmetry)

$= 2x0.0013 = 0.0026$

d) $P[|Z| < 2] = P[-2 < Z > 2] = P\left[Z < 2\right] - P\left[Z < -2\right] = 0.9772 - 0.0228 = \mathbf{0.9544}$

Alternatively, we can calculate

$P[|Z| < 2] = 2P[Z < -2]$ (as in part (b))

$= 2x0.0228 = 0.0456$

Hence, $P[|Z| < 2] = 1 - 0.0456 = \mathbf{0.9544}$

Question 4

a) The standardized variable is $Z = \frac{X-499}{120}$

$$P[X > 600] = P\left[X > \frac{600 - 499}{120}\right] = P[X > 0.842] = 1 - 0.8046 = \mathbf{0.1954}$$

b) We first find the 90th percentile of the standard normal distribution and then convert it to the x scale. Indeed, observe that

$P[Z < 1.28] = 0.8997 \approx 0.90$

The standardized score $z = 1.28$ corresponds to

$$x = 499 + 120(1.28) = \mathbf{652.6}$$

c) $P[X < 400] = P[Z < -0.83] = \mathbf{0.2033}$