

Exercise IV – Sampling and Sampling Distributions

1. A population consists of the four numbers $\{0, 2, 4, 6\}$. Consider drawing a random sample of size 2 with replacement.
 - a) List all possible samples and evaluate $\bar{x}$ for each.
 - b) Determine the sampling distribution of $\bar{X}$
 - c) Write down the population distribution and calculate its mean μ and standard deviation σ .
 - d) Calculate the mean and standard deviation of the sampling distribution of $\bar{X}$, obtained in part (b), and verify that these agree with μ and $\sigma/2$, respectively.
2. A random sample of size 150 is taken from the population of the ages of juniors enrolled at a large university during one semester. This population has mean 21.1 years and standard deviation 2.6. The population distribution is not normal.
 - a) Is it reasonable to assume a normal distribution for the sample mean $\bar{X}$? Why or why not?
 - b) Find the probability that $\bar{X}$ lies between 17.85 and 25.65 years.
 - c) Find the probability that $\bar{X}$ exceeds 25.91 years.
3. What sample size is required in order that the standard deviation of $\bar{X}$ be:
 - a) $1/4$ of the population standard deviation?
 - b) $1/7$ of the population standard deviation?
 - c) 12% of the population standard deviation?
4. Consider a random sample of 49 abrasion measurements.
 - a) Find the probability that the sample mean $\bar{X}$ will lie within 2 units of the population mean - that is $P[-2 \leq \bar{X} - \mu \leq 2]$.
 - b) Find the number k so that $P[-k \leq \bar{X} - \mu \leq k] = 0.90$
 - c) What is the probability that will differ from $\bar{X}$ by more than 4 units?