Solution – V

Question 1

Recall that the point estimate of μ is $\bar{x} = \frac{\sum x_i}{n}$ and the estimated standard error is $\frac{s}{\sqrt{n}}$,

where $s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$

a) $\bar{x} = \frac{3230.84}{38} = 85.02$

b) $S.E = \frac{s}{\sqrt{n}} = \frac{\sqrt{\frac{2028.35}{37}}}{\sqrt{38}} = 1.201$

c) 95% error margin is $z_{0.025}\frac{s}{\sqrt{n}} = (1.96)(1.201) = 2.354$

Question 2

a) The population mean μ is estimated by $\bar{x} = 126.9$. Estimated $S.E. = \frac{10.5}{\sqrt{55}} = 1.416.$ So, the approximate 95.4% error margin is given by 2 (Estimated $S.E.$) $= 2(1.416) \approx 2.8.$

b) Observe that $1 - \alpha = 0.90$ implies $\alpha = 0.10$, so that $z_{\alpha/2} = z_{0.05} = 1.645$. A 90% confidence interval for μ is calculated as follows:

$\bar{x} \pm 1.645\frac{s}{\sqrt{n}} = 126.9 \pm 1.645(1.416) = 126.9 \pm 2.329$ or $(124.571, 129.228)$

Question 3

The alternative hypothesis H_1 is the assertion that is to be established; its opposite is the null hypothesis H_0.

a) Let μ denote the population mean mileage. The hypotheses are:
$H_0 : \mu = 50$, $H_1 : \mu < 50$

b) Let μ denote the population mean number of pages per transmission. The hypotheses are: $H_0 : \mu = 3.4$, $H_1 : \mu > 3.4$

c) Let p denote the probability of success with the method. The hypotheses are:
$H_0 : p = 0.5$, $H_1 : p > 0.5$

d) Let μ denote the mean fill. The hypotheses are: $H_0 : \mu = 16$, $H_1 : \mu \neq 16$

e) Let μ denote the mean percent fat content. The hypotheses are:
$H_0 : p = 0.04$, $H_1 : p > 0.04$

Question 4

Let μ denote the mean number of words per sentence.

a) We test the hypotheses: $H_0 : \mu = 9.1$, $H_1 : \mu \neq 9.1$

b) The test statistic is $z = \frac{\bar{x}-9.1}{\frac{s}{\sqrt{n}}}$

c) Since H_1 is two-sided, the rejection region $R: |z| \geq z_{\alpha/2}$

d) Using x = 8.6, s =1.2, n = 36 , we see that the test statistic is $z = \frac{8.6-9.1}{\frac{1.2}{\sqrt{36}}} = -2.5$. For $\alpha = 0.10$, the rejection region is $R: |z| \geq z_{0.05} = 1.645$. Since the test statistic value is in R, we reject H_0 at $\alpha = 0.10$.

e) The associated p-value is $2P(Z \leq -2.5) = 2(0.0062) = 0.0124$.

f) Since we rejected H_0, we could have made a Type I error in that we rejected the fact that there are 9.1 words per sentence, on average, when this is in fact the case.