



MAT001

Forkurs i matematikk

Løsninger 2018

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Løsning / fasit på utvalgte oppgaver.

Dersom det ikke er løsning på en oppgave du har gjort, sammenlign ditt svar med svaret til en medstudent. Eller spør foreleser.

FASIT FoMA 1-5

101

a) $x^2 - 2x$

b) $x^2 - 5x + 6$

c) $2a^2 + ab - b^2$

d) $-x^3 + 1$

e) $-a^4 + 1$

102

a) $x^2 + 4x + 4$

b) $25 + 10a + a^2$

c) $9b^2 - 12b + 4$

d) $25x^2 - 1$

e) $9x^2 + 24xy + 16y^2$

f) $25 - 30x + 9x^2$

g) $x^4 + 2x^2 + 1$

h) $a^2 - \frac{2a}{b} + \frac{1}{b^2}$

i) $1 - x^4$

j) $a^2b^2 - 2ab + 1$

103

a) $a(4a - b)$

b) $5x(x - y)$

c) $3x^2y(y^2 - 3)$

d) $(x + 2)^2$

e) $(x - 4)^2$

f) $2(x + 4y)^2$

g) $(3x - 5)(3x + 5)$

h) $(6 - 9y)(6 + 9y)$

i) $(x^2 + 1)(x + 1)(x - 1)$

104

a) $\frac{a}{4b}$

b) $a + b$

c) $\frac{4(x-3)^2}{4(x+3)(x-3)} = \frac{x-3}{x+3}$

d) $\frac{a(2+a)}{2-a} = \frac{2a+a^2}{2-a}$

201

- a) 16
- b) $\frac{27}{64}$
- c) 16
- d) 0,008
- e) -1
- f) 1
- g) -1
- h) -1

202

- a) 6^4
- b) $(-3)^3$
- c) $(\frac{1}{2})^4$
- d) 3^4
- e) 5^4
- f) $\frac{1}{2^7} = (\frac{1}{2})^7$

205

- a) $2^8 = 256$
- b) $3^6 = 729$
- c) $4^{-1} = 2^{-2} = \frac{1}{4}$
- d) $2^{-6} = (\frac{1}{2})^6 = \frac{1}{64}$
- e) $2^6 = 64$
- f) a^5
- g) a^{12}
- h) $x^{-1} = \frac{1}{x}$

207

- a) $8a^3$
- b) $a^{12}b^8$
- c) a^8b^2

208

- a) $\frac{3^4}{2^4} = \frac{81}{16}$
- b) $10^2 = 100$
- c) x^3
- d) y^2

209

- a) 230000
- b) 0,000053
- c) 40000

210

- a) $6 \cdot 10^{-4}$
- b) $1,25 \cdot 10^6$
- c) $7,9 \cdot 10^{-5}$
- d) $4,06 \cdot 10^8$
- e) $3,14 \cdot 10^{10}$

213

- a) 6
- b) 9
- c) $9\sqrt{3}$
- d) 2
- e) $\sqrt{2^3} = 2\sqrt{2}$
- f) 0,1
- g) 0,8
- h) $\frac{1}{4}$
- i) $2^{-3} = \frac{1}{8}$

214

- a) $\frac{4\sqrt{5}}{5}$
- b) $\frac{\sqrt{6}}{2}$
- c) $\frac{4}{3}$

215

- a) =
- b) $\neq$
- c) $\neq$
- d) $\neq$

216

- a) $7 + 2\sqrt{10}$
- b) $7 - 2\sqrt{10}$
- c) 3

217

- a) $1 - \sqrt{15}$
- b) $5\sqrt{3} - 9$

301

- a, 8
- b, -1
- c, -1
- d, 2
- e, 0
- f, $-\frac{18}{19}$

- g, $\frac{8}{45}$
- h, -2

305

- a, $x=2, y=0$
- b, $x=\frac{2}{5}, y=-\frac{2}{5}$
- c, alle x og y som passer
i $x-2y=3$ er løsning
- d, ingen løsning

306

- a, $x=3, y=2$
- b, $x=\frac{44}{9}, y=-\frac{14}{9}$

403

- a, $x=2, x=3$
- b, $x=-7, x=2$
- c, $x=\frac{1}{3}, x=-3$
- d, $x=-1, x=-\frac{3}{4}$
- e, $x=\frac{1}{2}$
- f, ingen løsning
- g, ingen løsning

409

- a, $x > 7$
- b, $x < 7$
- c, $x < 2$
- d, $x \geq 2$
- e, $x < -1$

410

- a, $\langle \leftarrow, -4 \rangle \cup \langle 3, \rightarrow \rangle$
- b, $\langle 1, \frac{3}{2} \rangle$
- c, $\langle -3, 2 \rangle$
- d, $\langle \leftarrow, \frac{2}{5} \rangle \cup \langle 1, \rightarrow \rangle$
- e, $\langle -5, 2 \rangle$
- f, $\langle \frac{1}{3}, 1 \rangle$
- g, $\langle \leftarrow, -\frac{1}{2} \rangle \cup \langle 1, \rightarrow \rangle$
- h, $\langle -2, \rightarrow \rangle$
- i, $\langle \leftarrow, -1 \rangle \cup \langle -\frac{1}{2}, \rightarrow \rangle$

412

- a, $\langle \leftarrow, -1 \rangle \cup \langle 4, \rightarrow \rangle$
- b, $[-1, 2]$
- c, $\langle -4, -1 \rangle$
- d, $\langle 2, 4 \rangle$
- e, $\langle -1, -\frac{1}{2} \rangle \cup \langle 2, \rightarrow \rangle$

413

$$\langle -4, \rightarrow \rangle$$

501

- a) $x^2 - 3x + 1$
- b) $x - 2$
- c) $2x + \frac{7}{2}$
- d) $x^2 + 4$
- e) $x^2 + x + 1$
- f) $x^3 - 4x^2 + 21x - 84$
- rest 340

502

- a) $(x+2)(x+1)(x-1)$
- b) $(x-1)(x+3)(x-3)$
- c) $x(x+1)^2(x-2)$
- d) $(x+1)(x-1)(x^2+1)$
- e) $(x^2+4)(x+1)(x-1)$
- f) $(x^2+3x+4)(x+2)$
- g) $-(x^2+x+1)(x-1)$

503

- a) $\{\pm 2\}$
- b) $\{-5, 1\}$
- c) $\{\frac{1}{2}, \frac{3}{2}\}$
- d) $\{0, 2\}$

504

- a) $\langle 0, 2 \rangle$
- b) $\langle \leftarrow, 3 \rangle \cup \langle 1, \rightarrow \rangle$
- c) $\langle -\frac{1}{2}, \frac{3}{2} \rangle$
- d) $\langle \leftarrow, -5 \rangle \cup [1, \rightarrow \rangle$

FOMA 6-8 egner seg ikke som fasit.

Løsningsforslag på de neste sidene for oppgavene:

603 c

607 / 608

705

801 c

802 d

FoMa 6 (løsning)

603C

$$f(x) = -x^2 - x + 6$$

x	0	1	2	3	-1/2	-1	2	3	-4
f(x)	6	5	0	-6	25/4	6	4	0	-6

$$g(x) = x^2 - 4x + 3$$

x	0	1	2	3	4	5	-1	3/2
g(x)	3	0	-1	0	3	8	8	-2/4

1) Skjæringspunkt

$$\text{Aulest: } (2.2, -1), (-0.7, 6.2)$$

Ved regning:

*

$$g(x) = f(x)$$

$$x^2 - 4x + 3 = -x^2 - x + 6$$

$$2x^2 - 3x - 3 = 0$$

$$x = \frac{3 \pm \sqrt{9 + 24}}{4}$$

$$x = \frac{3 \pm \sqrt{33}}{4} \approx \begin{cases} 2.19 \\ -0.69 \end{cases}$$

2) $f(x) > g(x)$

$$-x^2 - x + 6 > x^2 - 4x + 3$$

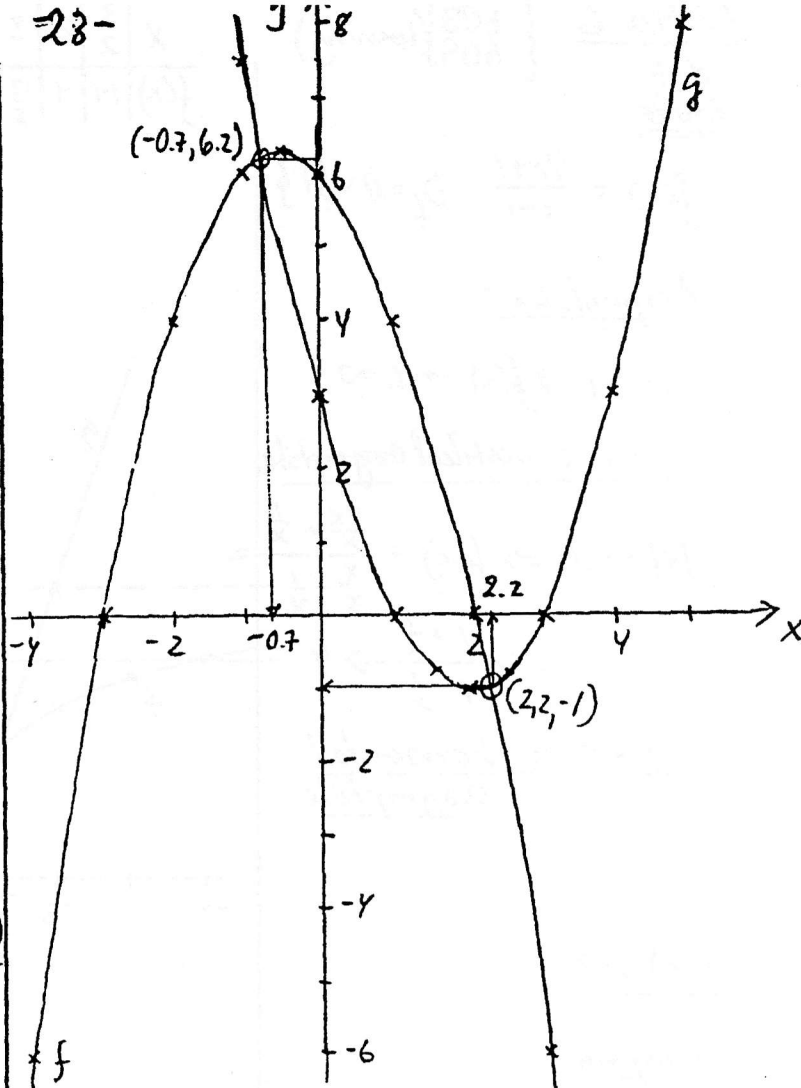
$$-2x^2 + 3x + 3 > 0$$

$$2x^2 - 3x - 3 < 0 \text{ se ovenfor *}$$

$$2\left(x - \frac{3 + \sqrt{33}}{4}\right)\left(x - \frac{3 - \sqrt{33}}{4}\right) < 0$$

$$2(x - 2.19)(x + 0.69) < 0$$

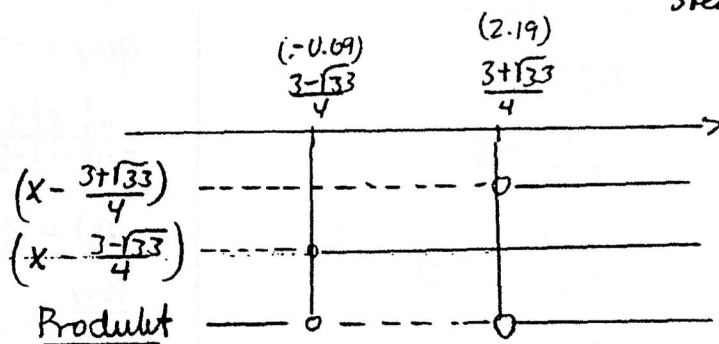
23-



$$f(2.19) = -(2.19^2) - 2.19 + 6 = -0.99 \text{ dvs } (2.19, -0.99)$$

$$f(-0.69) = -(-0.69^2) - (-0.69) + 6 = 6.21 \text{ dvs } (-0.69, 6.21)$$

(kontrollen stemmer)



$$f(x) > g(x) \quad L = \left\langle \frac{3 - \sqrt{33}}{4}, \frac{3 + \sqrt{33}}{4} \right\rangle$$

Kontroll på grafen: f ligger over g når $x \in \langle -0.7, 2.2 \rangle$ dette stemmer med regningen

Fo Ma 6 } 608 (løsning)
~~607~~
~~608~~

$$f(x) = \frac{4x+1}{x-1} \quad D_f = \mathbb{R} \setminus \{1\}$$

Asymptotes:

$$x \rightarrow 1 \Rightarrow f(x) \rightarrow \pm \infty$$

$x=1$ er vertical asymptote

$$|x| \rightarrow \infty \Rightarrow f(x) = \frac{\frac{4x}{x} + \frac{1}{x}}{\frac{x}{x} - \frac{1}{x}} = \frac{4 + \frac{1}{x}}{1 - \frac{1}{x}} \rightarrow 4$$

$y=4$ er horizontal asymptote

$$f(x) > 3$$

Grafisk

Vi tegner $y=3$

Grafen til f ligger over $y=3$

Når $x \in \langle \leftarrow, -1 \rangle \cup \langle 1, \rightarrow \rangle$

Ved regning

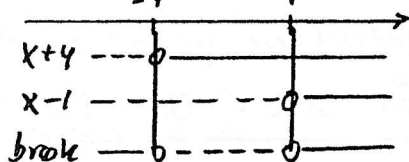
$$f(x) > 3$$

$$\frac{4x+1}{x-1} > 3$$

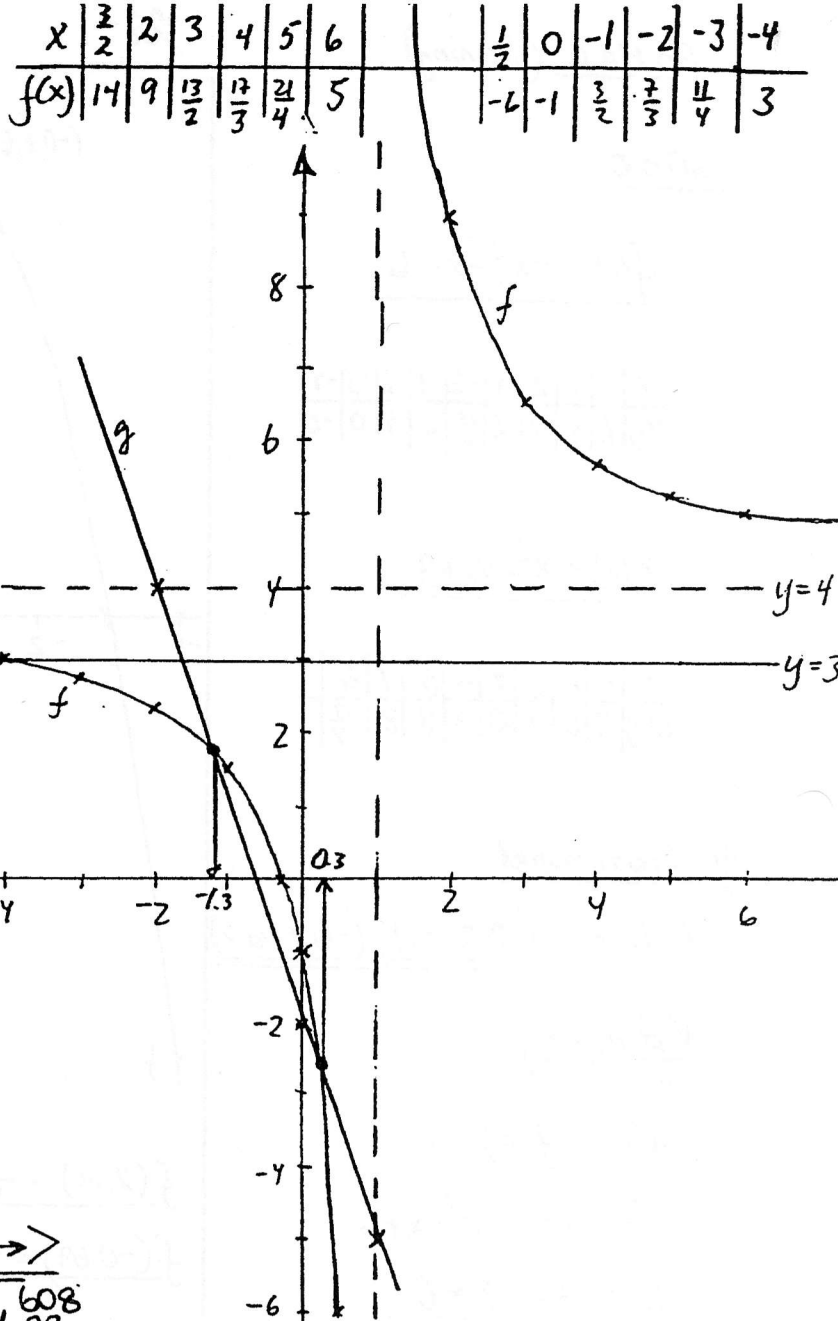
$$\frac{4x+1}{x-1} - 3 > 0$$

$$\frac{4x+1-3x+3}{x-1} > 0$$

$$\frac{x+4}{x-1} > 0$$



$$L = \langle \leftarrow, -4 \rangle \cup \langle 1, \rightarrow \rangle$$



~~608~~
~~609~~

$$g(x) = -3x-2$$

$$\frac{x}{g(x)} = \frac{-2}{4} = -\frac{1}{2}$$

$$f(x) = g(x)$$

$$\frac{4x+1}{x-1} = -3x-2 \quad | \cdot (x-1)$$

$$4x+1 = -3x^2+3x-2x+2$$

$$*) 3x^2+3x-1=0$$

$$x = \frac{-3 \pm \sqrt{9+12}}{6}$$

$$x = \frac{-3 \pm \sqrt{21}}{6} = \begin{cases} 0.26 \\ -1.26 \end{cases}$$

Dette stemmer m. grafen

$$f(x) > g(x) \text{ grafisk}$$

f ligger over g når

$$x \in \langle -1.3, 0.3 \rangle \cup \langle 1, \rightarrow \rangle$$

$$f(x) > g(x)$$

$$\frac{4x+1}{x-1} > -3x-2$$

$$\frac{4x+1}{x-1} + 3x+2 > 0$$

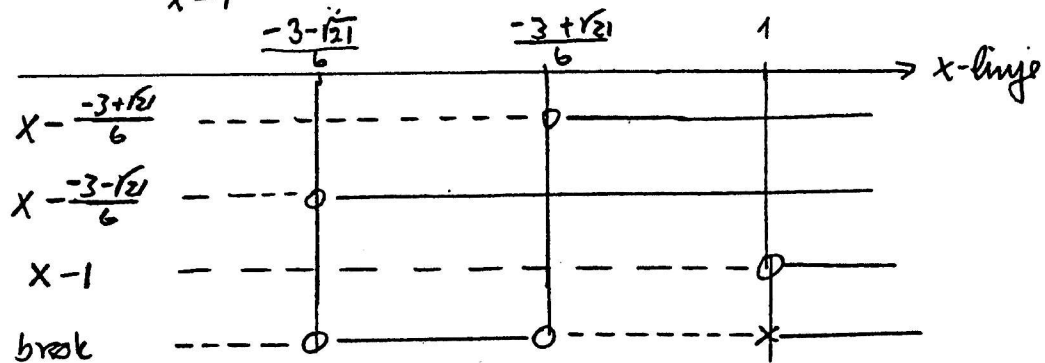
$$\frac{3x^2+3x-1}{x-1} > 0 \text{ se } *)$$

$$\frac{3(x - \frac{-3+\sqrt{21}}{6})(x - \frac{-3-\sqrt{21}}{6})}{x-1} > 0$$

oppgave 8 forts.

$$f(x) > g(x)$$

$$\frac{3(x - \frac{-3+\sqrt{21}}{6})(x - \frac{-3-\sqrt{21}}{6})}{x-1} > 0$$



$$h = \left(\frac{-3-\sqrt{21}}{6}, \frac{-3+\sqrt{21}}{6} \right) \cup \left(1, \rightarrow \right)$$

$$= \left(-1.26, 0.26 \right) \cup \left(1, \rightarrow \right)$$

205.

$$f(x) = \frac{2x-2}{x}, \quad x \neq 0$$

Asymptoter: $x=0$
 $y=2$

$$g(x) = 2x + a$$

A) $f(x) = 2x + a$

$$\frac{2x-2}{x} = 2x + a$$

$$2x-2 = 2x^2 + ax$$

$$2x^2 - (2-a)x + 2 = 0$$

$$x = \frac{2-a \pm \sqrt{(2-a)^2 - 16}}{4}$$

$$x = \frac{2-a \pm \sqrt{a^2 - 4a - 12}}{4}$$

her må $a^2 - 4a - 12 \geq 0$

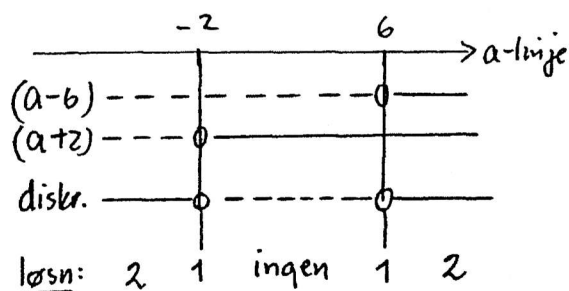
Vi løser $a^2 - 4a - 12 = 0$

$$a = \frac{4 \pm \sqrt{16 + 48}}{2}$$

$$a = 6 \vee a = -2$$

Altså: $a^2 - 4a - 12 \geq 0$

$$(a-6)(a+2) \geq 0$$

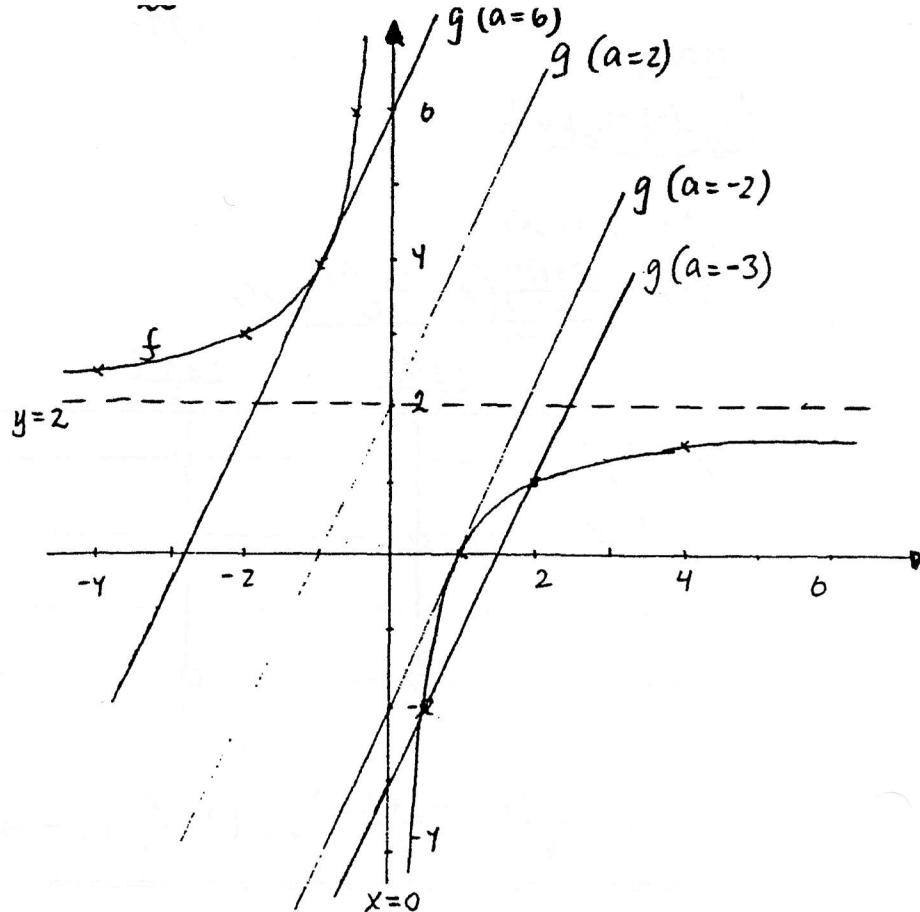


$$a < -2 \vee a > 6 : 2 \text{ løsninger } L = \left\{ \frac{2-a \pm \sqrt{a^2 - 4a - 12}}{4} \right\}$$

$$a = -2 : 1 \text{ løsning } L = \{1\}$$

$$a = 6 : 1 \text{ løsning } L = \{-1\}$$

$$-2 < a < 6 : \text{ingen løsning } L = \emptyset$$



b) geometrisk tolkning

Løsning av ligningen $f(x) = 2x + a$ betyr grafisk å bestemme de x-verdier som gir skjæring mellom grafen til f og grafen til g.

Av figuren ovenfor ser vi:

$\left. \begin{matrix} a < -2 \\ \text{eller} \\ a > 6 \end{matrix} \right\} \Rightarrow 2 \text{ skj.-pkt} \\ \text{dvs. 2 løsninger på ligningen}$

$\left. \begin{matrix} a = -2 \\ \text{eller} \\ a = 6 \end{matrix} \right\} \Rightarrow \text{tangering (ett felleopkt)} \\ \text{dvs. 1 løsning}$

$-2 < a < 6 \Rightarrow \text{ingen skjæring} \\ \text{dvs. ingen løsn. på lign.}$

801c

$$f(x) = x^3 - 2x^2 - 5x + 6$$

Nullpkt.

$$f(1) = 0 \Rightarrow f(x) : (x-1) \text{ går opp.}$$

$$(x^3 - 2x^2 - 5x + 6) : (x-1) = x^2 - x - 6$$

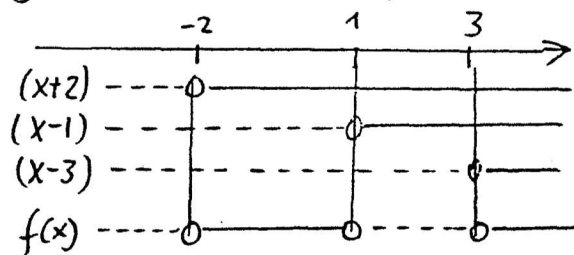
$$\begin{array}{r} -(x^3 - x^2) \\ \hline -x^2 - 5x \\ -(-x^2 + x) \\ \hline -6x + 6 \\ -(-6x + 6) \\ \hline 0 \end{array}$$

$$x^2 - x - 6 = 0$$

$$x = \frac{1 \pm \sqrt{1+24}}{2} = \begin{cases} 3 \\ -2 \end{cases}$$

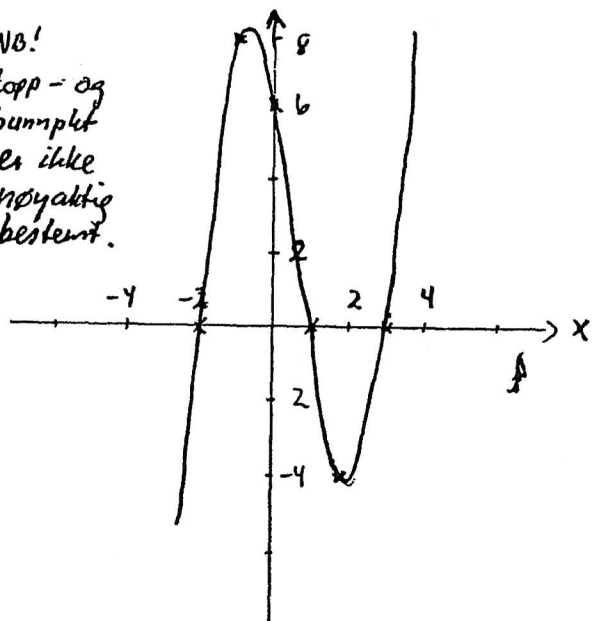
Nullpunktene blir $x = -2$
 $x = 1$
 $x = 3$

$$f(x) = (x+2)(x-1)(x-3)$$



$$f(x) > 0 \text{ når } x \in \langle -2, 1 \rangle \cup \langle 3, \infty \rangle$$

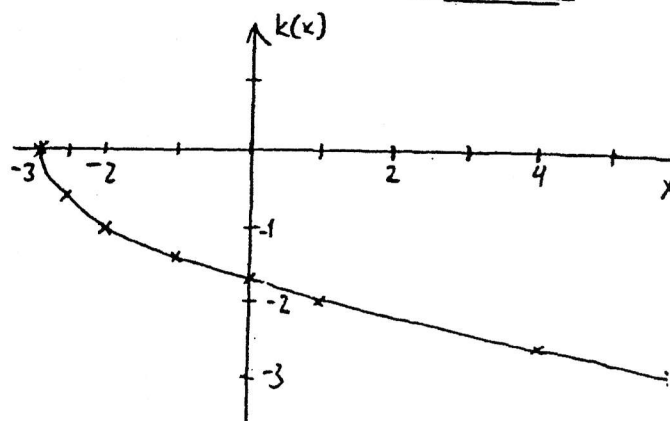
NB!
 topp- og
 bunnpkt
 er ikke
 nøyaktig
 bestemt.



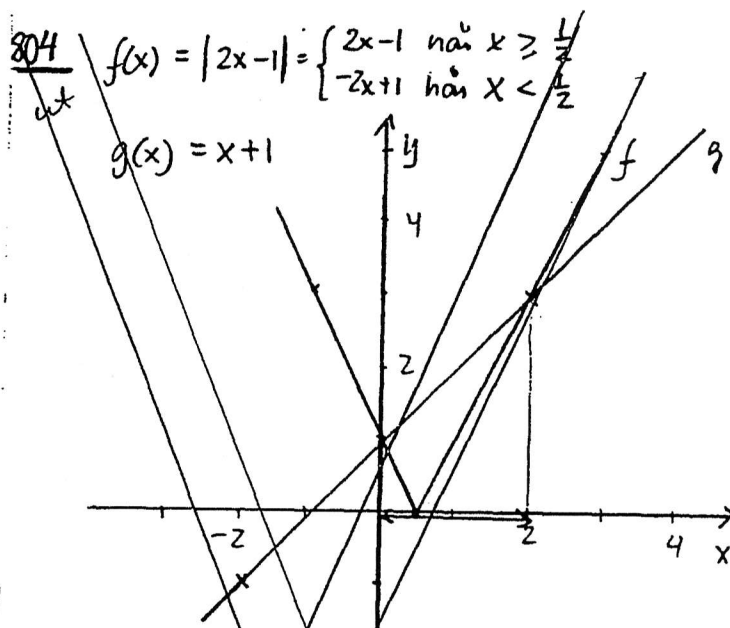
802d

$$k(x) = -\sqrt{x+3} \text{ her må } x+3 \geq 0 \\ x \geq -3$$

Største def. mengde $[-3, \infty)$



$$V_f = \langle -1, 0 \rangle$$



a) $f(x) = g(x)$

1) $x \geq \frac{1}{2} \Rightarrow$

$$2x-1 = x+1 \\ x = 2 \text{ (kan brukes)}$$

2) $x < \frac{1}{2} \Rightarrow$

$$\begin{aligned} -2x+1 &= x+1 \\ -3x &= 0 \\ x &= 0 \text{ (kan brukes)} \end{aligned}$$

$$L = \{0, 2\}$$

b) $f(x) < g(x)$

1) $x \geq \frac{1}{2} \Rightarrow$

$$\begin{aligned} 2x-1 &< x+1 \\ x &< 2, L_1 = [\frac{1}{2}, 2) \end{aligned}$$

2) $x < \frac{1}{2} \Rightarrow$

$$\begin{aligned} -2x+1 &< x+1 \\ -3x &< 0 \\ x &> 0, L_2 = (0, \frac{1}{2}) \end{aligned}$$

$$L = \langle 0, 2 \rangle$$

Grafisk: f skjærer g for $x=0$ og for $x=2$
 f ligger under g (dvs. $f(x) < g(x)$)
 når $x \in \langle 0, 2 \rangle$

Regningen stemmer med grafen.